Squaring the Eqs.(4) and (7) and comparing the result, we find the following correlation between gravitational energy and *momentum* :

$$\frac{E_g^2}{c^2}=p^2+m_g{}^2c^2. \qquad (19)$$

The energy expressed as a function of the *momentum* is, as we know, called *Hamiltonian* or Hamilton's function:

$$H_g=c\sqrt{p^2+m_g{}^2c^2}. \qquad (20)$$

Let us now consider the problem of quantization of gravity. Clearly there is something unsatisfactory about the whole notion of quantization. It is important to bear in mind that the quantization process is a series of rules-of-thumb rather than a well-defined algorithm, and contains many ambiguities. In fact, for electromagnetism we find that there are (at least) two different approaches to quantization and that while they appear to give the same theory they may lead us to very different quantum theories of gravity. Here we will follow a new theoretical strategy: It is known that starting from the Schrödinger equation we may obtain the well-known expression for energy of a particle in periodic motion inside a cubical box of edge length L [7]. The result now is

$$E_n=\frac{n^2h^2}{8m_gL^2} \qquad n=1,2,3,... \qquad (21)$$

Note that the term $h^2/8m_gL^2$ (energy) will be minimum for $L=L_{max}$ where L_{max} is the maximum edge length of a cubical box whose maximum diameter

$$d_{max}=L_{max}\sqrt{3} \qquad (22)$$

is equal to *the maximum* length scale *of the Universe*.

The minimum energy of a particle is obviously its inertial energy at rest $m_gc^2=m_ic^2$. Therefore we can write

$$\frac{n^2h^2}{8m_gL_{max}^2}=m_gc^2$$

Then from the equation above follows that

$$m_g=\pm\frac{nh}{cL_{max}\sqrt{8}} \qquad (23)$$

whence we see that there is a *minimum value* for m_g given by

$$m_{g(min)}=\pm\frac{h}{cL_{max}\sqrt{8}} \qquad (24)$$

The *relativistic* gravitational mass $M_g=\left|m_g\left(1-V^2/c^2\right)^{-1/2}\right|$, defined in the Eqs.(4), shows that

$$M_{g(min)}=\left|m_{g(min)}\right| \qquad (25)$$

The *box normalization* leads to the conclusion that the *propagation number* $k=\left|\vec{k}\right|=2\pi/\lambda$ is restricted to the values $k=2\pi n/L$. This is deduced assuming an *arbitrarily large but finite* cubical box of volume L^3 [8]. Thus, we have

$$L=n\lambda$$

From this equation, we conclude that

$$n_{max}=\frac{L_{max}}{\lambda_{min}}$$

and

$$L_{min}=n_{min}\lambda_{min}=\lambda_{min}$$

Since $n_{min}=1$. Therefore we can write that

$$L_{max}=n_{max}L_{min} \qquad (26)$$

From this equation we thus conclude that

$$L = nL_{min} \tag{27}$$

or

$$L = \frac{L_{max}}{n} \tag{28}$$

Multiplying (27) and (28) by $\sqrt{3}$ and reminding that $d = L\sqrt{3}$, we obtain

$$d = nd_{min} \quad or \quad d = \frac{d_{max}}{n} \tag{29}$$

Equations above show that the length (and therefore the *space*) is *quantized*.

By analogy to (23) we can also conclude that

$$M_{g(\boldsymbol{max})} = \frac{n_{\boldsymbol{max}}h}{cL_{\boldsymbol{min}}\sqrt{8}} \tag{30}$$

since the relativistic gravitational mass, $M_g = \left| m_g \left(1 - V^2/c^2\right)^{-\frac{1}{2}} \right|$, is just a multiple of m_g.

Equation (26) tells us that $L_{min} = L_{max}/n_{max}$. Thus Eq.(30) can be rewritten as follows

$$M_{g(\boldsymbol{max})} = \frac{n_{\boldsymbol{max}}^2 h}{cL_{\boldsymbol{max}}\sqrt{8}} \tag{31}$$

Comparison of (31) with (24) shows that

$$M_{g(max)} = n_{max}^2 \left| m_{g(min)} \right| \tag{32}$$

which leads to following conclusion that

$$M_g = n^2 \left| m_{g(min)} \right| \tag{33}$$

This equation shows that *the gravitational mass is quantized*.

Substitution of (33) into (13) leads to quantization of *gravity*, i.e.,

$$g = -\frac{GM_g}{r^2} = n^2\left(-\frac{G\left|m_{g(min)}\right|}{\left(r_{max}/n\right)^2} \right) =$$
$$= n^4 g_{min} \tag{34}$$

From the Hubble's law follows that

$$V_{max} = \tilde{H}l_{max} = \tilde{H}\left(d_{max}/2\right)$$

$$V_{min} = \tilde{H}l_{min} = \tilde{H}\left(d_{min}/2\right)$$

whence

$$\frac{V_{max}}{V_{min}} = \frac{d_{max}}{d_{min}}$$

Equations (29) tell us that $d_{max}/d_{min} = n_{max}$. Thus the equation above gives

$$V_{min} = \frac{V_{max}}{n_{max}} \tag{35}$$

which leads to following conclusion

$$V = \frac{V_{max}}{n} \tag{36}$$

this equation shows that *velocity* is also quantized.

From this equation one concludes that we can have $V = V_{max}$ or $V = V_{max}/2$, but there is nothing in between. This shows clearly that V_{max} cannot be equal to c (speed of light in vacuum). Thus, it follows that

$$
\begin{array}{lll}
n = 1 & V = V_{max} & \\
n = 2 & V = V_{max}/2 & \\
n = 3 & V = V_{max}/3 & \textit{Tachyons} \\
\ldots\ldots & \ldots\ldots\ldots\ldots & \\
n = n_x - 1 & V = V_{max}/(n_x - 1) & \\
\hline
n = n_x & V = V_{max}/n_x = c \quad \leftarrow & \\
n = n_x + 1 & V = V_{max}/(n_x + 1) & \textit{Tardyons} \\
n = n_x + 2 & V = V_{max}/(n_x + 2) & \\
\ldots\ldots & \ldots\ldots\ldots\ldots &
\end{array}
$$

where n_x *is a big number.*

Then c is the speed *upper limit* of the *Tardyons* and also the speed *lower limit* of the *Tachyons*. Obviously, this limit is *always the same in all inertial frames*. Therefore c can be used as a *reference speed*, to which we may compare any speed V, as occurs in the relativistic factor $\sqrt{1-V^2/c^2}$. Thus, in this factor, c

does not refer to maximum propagation speed of the interactions such as some authors suggest; c is just a speed limit which remains the same in any inertial frame.

The temporal coordinate x^0 of space-time is now $x^0 = V_{max} t$ ($x^0 = ct$ is then obtained when $V_{max} \to c$). Substitution of $V_{max} = nV = n(\tilde{H}l)$ into this equation yields $t = x^0/V_{max} = (1/n\tilde{H})(x^0/l)$. On the other hand, since $V = \tilde{H}l$ and $V = V_{max}/n$ we can write that $l = V_{max}\tilde{H}^{-1}/n$. Thus $(x^0/l) = \tilde{H}(nt) = \tilde{H}t_{max}$. Therefore we can finally write

$$t = (1/n\tilde{H})(x^0/l) = t_{max}/n \qquad (37)$$

which shows the quantization of *time*.

From Eqs.(27) and (37) we can easily conclude that the *spacetime is not continuous* it is *quantized*.

Now, let us go back to Eq. (20) which will be called the *gravitational* Hamiltonian to distinguish it from the *inertial* Hamiltonian H_i:

$$H_i = c\sqrt{p^2 + m_{i0}^2 c^2}. \qquad (38)$$

Consequently, the Eq. (18) can be rewritten in the following form:

$$H_i - H_g = 2\Delta H_i \qquad (39)$$

where ΔH_i is the *variation on the inertial Hamiltonian* or *inertial kinetic energy*. A *momentum* variation Δp yields a variation ΔH_i given by:

$$\Delta H_i = \sqrt{(p+\Delta p)^2 c^2 + m_{i0}^2 c^4} - \sqrt{p^2c^2 + m_{i0}^2 c^4} \qquad (40)$$

By considering that the particle is *initially at rest* $(p = 0)$. Then Eqs. (20), (38) and (39) give respectively: $H_g = m_g c^2$, $H_i = m_{i0}c^2$ and

$$\Delta H_i = \left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right] m_{i0}c^2$$

By substituting H_g, H_i and ΔH_i into Eq.(39) we get

$$m_g = m_{i0} - 2\left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right] m_{i0}. \qquad (41)$$

This is the *general expression of correlation between the gravitational and inertial mass.* Note that for $\Delta p > m_{i0}c\left(\sqrt{5}/2\right)$, the value of m_g becomes *negative*.

Equation (41) shows that m_g decreases of Δm_g for an increase of Δp. Thus, starting from (4) we obtain

$$p + \Delta p = \frac{(m_g - \Delta m_g)\, V}{\sqrt{1 - (V/c)^2}}$$

By considering that the particle is *initially at rest* $(p = 0)$, the equation above gives

$$\Delta p = \frac{(m_g - \Delta m_g)\, V}{\sqrt{1 - (V/c)^2}}$$

From the Eq.(16) we obtain: $E_g = 2E_{i0} - E_i = 2E_{i0} - (E_{i0} + \Delta E_i) = E_{i0} - \Delta E_i$

However, Eq.(14) tells us that $-\Delta E_i = \Delta E_g$; what leads to $E_g = E_{i0} + \Delta E_g$ or $m_g = m_{i0} + \Delta m_g$. Thus, in the expression of Δp we can replace $(m_g - \Delta m_g)$ by m_{i0}, i.e.,

$$\Delta p = \frac{m_{i0}\, V}{\sqrt{1 - (V/c)^2}}$$

We can therefore write

$$\frac{\Delta p}{m_{i0}c} = \frac{V/c}{\sqrt{1 - (V/c)^2}} \qquad (42)$$

By substitution of the expression above into Eq. (41) we thus obtain:

$$m_g = m_{i0} - 2\left[\left(1 - V^2/c^2\right)^{-\frac{1}{2}} - 1\right] m_{i0} \qquad (43)$$

For $V = 0$ we obtain $m_g = m_{i0}$. Then

$$m_{g(min)} = m_{i0(min)}$$

Substitution of $m_{g(min)}$ into the *quantized* expression of M_g (Eq. (33)) gives

$$M_g = n^2 m_{i0(min)}$$

Where $m_{i0(min)}$ is the *elementary*